25th Feb. 2021 | Shift - 1
MATHEMATICS

JEE I NEET I Foundation

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SELECTIONS SINCE 2007

Section : Mathematics Section A

1. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is :

(1) $\frac{1}{54}$

(2) $\frac{1}{72}$

(3) $\frac{1}{36}$

(4) $\frac{5}{216}$

Ans. (4)

Sol. $ax^2 + bx + c = 0$

$a, b, c \in \{1,2,3,4,5,6\}$

$n(s) = 6 \times 6 \times 6 = 216$

$D = 0 \Rightarrow b^2 = 4ac$

$ac = \frac{b^2}{4}$ If $b = 2, ac = 1 \Rightarrow a = 1, c = 1$

If $b = 4, ac = 4 \Rightarrow a = 1, c = 4$

$a = 4, c = 1$

$a = 2, c = 2$

If $b = 6, ac = 9 \Rightarrow a = 3, c = 3$

$\therefore \text{probability} = \frac{5}{216}$

2. Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is :

(1) $\frac{3}{4}$

(2) $\frac{1}{2}$

(3) $\frac{5}{8}$

(4) $\frac{3}{8}$

Ans. (3)

Sol. $l^2 + m^2 + n^2 = 1$

$\therefore 2n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

$\therefore l^2 + m^2 = \frac{1}{2}$ & $l + m = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{2} - 2lm = \frac{1}{2}$

$\Rightarrow l\ m = 0$ or $m = 0$

$\therefore l = 0,\ m = \frac{1}{\sqrt{2}}$ or $l = \frac{1}{\sqrt{2}}$

$< 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ or $< \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} >$

$\therefore \cos\alpha = 0 + 0 + \frac{1}{2} = \frac{1}{2}$

$\therefore \sin^4\alpha + \cos^4\alpha = 1 - \frac{1}{2}\sin^2(2\alpha) = 1 - \frac{1}{2}, \frac{3}{4} = \frac{5}{8}$

3. The value of the integral

$\int \frac{\sin\theta.\sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta$ is

(where c is a constant of integration)

(1) $\frac{1}{18}\left[9 - 2\sin^6\theta - 3\sin^4\theta - 6\sin^2\theta\right]^{\frac{3}{2}} + c$

(2) $\frac{1}{18}\left[11 - 18\sin^2\theta + 9\sin^4\theta - 2\sin^6\theta\right]^{\frac{3}{2}} + c$

(3) $\frac{1}{18}\left[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta\right]^{\frac{3}{2}} + c$

(4) $\frac{1}{18}\left[9 - 2\cos^6\theta - 3\cos^4\theta - 6\cos^2\theta\right]^{\frac{3}{2}} + c$

Ans. (3)

Sol. $\int \frac{2\sin^2\theta\cos\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{2\sin^2\theta} d\theta$

Let $\sin\theta = t$, $\cos\theta\ d\theta = dt$

$= \int (t^6 + t^4 + t^2)\sqrt{2t^4 + 3t^2 + 6}\, dt = \int (t^5 + t^3 + t)\sqrt{2t^6 + 3t^4 + 6t^2}\, dt$

Let $2t^6 + 3t^4 + 6t^2 = z$

$12(t^5 + t^3 + t)\ dt = dz$

$= \frac{1}{12}\int \sqrt{z}\, dz = \frac{1}{18} z^{3/2} + c$

$= \frac{1}{18}[(2\sin^6\theta + 3\sin^4\theta + 6\sin^2\theta)^{3/2} + C$

$= \frac{1}{18}\left[\left(1 - \cos^2\theta\right)\left(2(1 - \cos^2\theta)^2 + 3 - 3\cos^2\theta + 6\right)\right]^{3/2} + C$

$= \frac{1}{18}[(1 - \cos^2\theta)(2\cos^4\theta - 7\cos^2\theta + 11)]^{3/2} + C$

$= \frac{1}{18}[-2\cos^6\theta + 9\cos^4\theta - 18\cos^2\theta + 11]^{3/2} + C$

4. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is :

(1) $10(\sqrt{3}-1)$

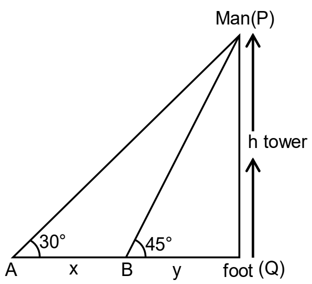
(2) $10\sqrt{3}$

(3) 10

(4) $10(\sqrt{3}+1)$

Ans. (4)

Sol.



$$\frac{h}{x+y}=\tan 30°$$

x + y = $\sqrt{3}h$ ....(1)

Also

$$\frac{h}{y}=\tan 45°$$

h = y ....(2)

put in (1)

x + y = $\sqrt{3}y$

x = $(\sqrt{3}-1)y$

$$\frac{x}{20}='v'\text{speed}$$

$\therefore$ time taken to reach

Foot from B

$$\Rightarrow \frac{y}{V}$$

$$\Rightarrow \frac{x}{(\sqrt{3}-1).x}\times 20$$

$\Rightarrow 10(\sqrt{3}+1)$

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5. If $0 < \theta, \phi < \frac{\pi}{2}, x = \sum_{n=0}^{\infty} \cos^{2n}\theta,\ y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and

$z = \sum_{n=0}^{\infty} \cos^{2n}\theta.\sin^{2n}\phi$ then :

(1) xyz = 4
(2) xy – z = (x + y)z
(3) xy + yz + zx = z
(4) xy + z = (x +y)z

Ans. (4)

Sol. $x = 1 + \cos^2\theta + \ldots\ldots\infty$

$$x = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta} \qquad \ldots\ldots(1)$$

$y = 1 + \sin^2\phi + \ldots\ldots\infty$

$$y = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi} \qquad \ldots\ldots(2)$$

$$z = \frac{1}{1-\cos^2\theta.\sin^2\phi} = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)} = \frac{xy}{xy-(x-1)(y-1)}$$

xz + yz – z = xy
xy + z = (x + y)z

6. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is :

(1) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

(2) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

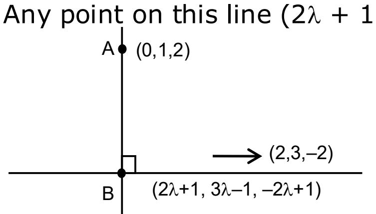
(3) $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

(4) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

Ans. (1)

Sol. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} = \lambda$

Any point on this line (2λ + 1, 3λ – 1, –2λ+1)



Direction ratio of given line (2, 3, –2)

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Direction ratio of line to be found $(2\lambda + 1, 3\lambda - 2, -2\lambda - 1)$

$\therefore\ \vec{d}_1.\vec{d}_2 = 0$

$\lambda = 2/17$

Direction ratio of line $(21, -28, -21) \equiv (3, -4, -3) \equiv (-3, 4, 3)$

7. The statement $A \to (B \to A)$ is equivalent to:

(1) $A \to (A \wedge B)$

(2) $A \to (A \vee B)$

(3) $A \to (A \to B)$

(4) $A \to (A \leftrightarrow B)$

Ans. (2)

Sol. $A \to (B \to A)$

$\Rightarrow A \to (\sim B \vee A)$

$\Rightarrow \sim A \vee (\sim B \vee A)$

$\Rightarrow \sim B \vee (\sim A \vee A)$

$\Rightarrow \sim B \vee t$

$= t$ (tantology)

From options :

(2) $A \to (A \vee B)$

$\Rightarrow \sim A \vee (A \vee B)$

$\Rightarrow (\sim A \vee A) \vee B$

$\Rightarrow t \vee B$

$\Rightarrow t$

8. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R is :

(1) 3

(2) 2

(3) 4

(4) 0

Ans. (1)

Sol. $D < 0$

$$\left(2\left(3k-1\right)\right)^2 - 4\left(8k^2 - 7\right) < 0$$

$$4\left(9k^2 - 6k + 1\right) - 4\left(8k^2 - 7\right) < 0$$

$k^2 - 6k + 8 < 0$

$(k-4)(k-2) < 0$

$2 < k < 4$

then $k = 3$

9. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?
(1) (0, 3)
(2) (−6, 0)
(3) (4, 5)
(4) (5, 4)

Ans. (4)

Sol. Equation of tangent : $y = mx + \frac{3}{2m}$

$m_T = \frac{1}{2}$ ($\because$ perpendicular to line $2x + y = 1$)

$\therefore$ tangent is : $y = \frac{x}{2} + 3 \Rightarrow x - 2y + 6 = 0$

10. Let f, g: N → N such that $f(n + 1) = f(n) + f(1)\ \forall n \in N$ and g be any arbitrary function. Which of the following statements is NOT true ?
(1) f is one-one
(2) If fog is one-one, then g is one-one
(3) If g is onto, then fog is one-one
(4) If f is onto, then $f(n) = n\ \forall n \in N$

Ans. (3)

Sol. $f(n + 1) = f(n) + 1$
$f(2) = 2f(1)$
$f(3) = 3f(1)$
$f(4) = 4f(1)$
….
$f(n) = nf(1)$
f(x) is one-one

11. Let the lines $(2 - i)z = (2 + i)\bar{z}$ and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is :

(1) $\frac{3}{\sqrt{2}}$

(2) $3\sqrt{2}$

(3) $\frac{3}{2\sqrt{2}}$

(4) $\frac{1}{2\sqrt{2}}$

Ans. (3)

Sol. $(2-i)z=(2+i)\bar{z}$

$\Rightarrow (2-i)(x+iy)=(2+i)(x-iy)$

$\Rightarrow 2x - ix + 2iy + y = 2x + ix - 2iy + y$

$\Rightarrow 2ix - 4iy = 0$

$L_1 : x - 2y = 0$

$\Rightarrow (2+i)z + (i-2)\bar{z} - 4i = 0.$

$\Rightarrow (2+i)(x+iy) + (i-2)(x-iy) - 4i = 0.$

$\Rightarrow 2x + ix + 2iy - y + ix - 2x + y + 2iy - 4i = 0$

$\Rightarrow 2ix + 4iy - 4i = 0$

$L_2 : x + 2y - 2 = 0$

Solve L_1 and L_2 $4y=2$, $y=\frac{1}{2}$

$\therefore x=1$

Centre $\left(1, \frac{1}{2}\right)$

$L_3 : iz + \bar{z} + 1 + i = 0$

$\Rightarrow i(x+iy) + x - iy + 1 + i = 0$

$\Rightarrow ix - y + x - iy + 1 + i = 0$

$\Rightarrow (x - y + 1) + i(x - y + 1) = 0$

Radius = distance from $\left(1, \frac{1}{2}\right)$ to $x - y + 1 = 0$

$$r = \frac{1 - \frac{1}{2} + 1}{\sqrt{2}}$$

$$r = \frac{3}{2\sqrt{2}}$$

12. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in:

(1) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(2) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

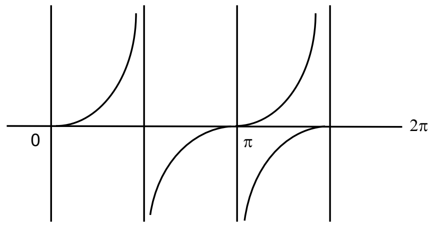
(3) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(4) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

y
1

Ans. (2)

Sol.



$\tan 2\theta\,(1 + \cos 2\theta) > 0$

$2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$

$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

13. The image of the point (3,5) in the line $x - y + 1 = 0$, lies on :

(1) $(x - 2)^2 + (y - 4)^2 = 4$

(2) $(x - 4)^2 + (y + 2)^2 = 16$

(3) $(x - 4)^2 + (y - 4)^2 = 8$

(4) $(x - 2)^2 + (y - 2)^2 = 12$

Ans. (1)

Sol. Image of P(3, 5) on the line $x - y + 1 = 0$ is

$\frac{x-3}{1} = \frac{y-5}{-1} = \frac{-2(3-5+1)}{2} = 1$

$x = 4,\ y = 4$

$\therefore$ Image is (4, 4)

Which lies on

$(x - 2)^2 + (y - 4)^2 = 4$

14. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4$, $x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to :

(1) (–5, 8)

(2) (5, 8)

(3) (5, –8)

(4) (–5, –8)

Ans. (2)

Sol. $f(1) = f(2)$

$\Rightarrow 1 - a + b - 4 = 8 - 4a + 2b - 4$

$3a - b = 7$ ...(1)

$f'(x) = 3x^2 - 2ax + b$

$\Rightarrow f'\left(\frac{4}{3}\right) = 0 \Rightarrow 3 \times \frac{16}{9} - \frac{8}{3}a + b = 0$

$\Rightarrow -8a + 3b = -16$ ...(2)

$a = 5,\ b = 8$

15. If the curves, $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ intersect each other at an angle of 90°, then which of the following relations is true ?

(1) a + b = c + d

(2) a − b = c − d

(3) ab = $\frac{c+d}{a+b}$

(4) a − c = b + d

Ans. (2)

Sol. $\frac{x^2}{a}+\frac{y^2}{b}=1$ ....(1)

diff : $\frac{2x}{a}+\frac{2y}{b}\frac{dy}{dx}=0 \Rightarrow \frac{y}{b}\frac{dy}{dx}=\frac{-x}{a}$

$\frac{dy}{dx}=\frac{-bx}{ay}$ ......(2)

$\frac{x^2}{c}+\frac{y^2}{d}=1$ ....(3)

Diff : $\frac{dy}{dx}=\frac{-dx}{cy}$ ....(4)

$m_1m_2 = -1 \Rightarrow \frac{-bx}{ay}\times\frac{-dx}{cy} = -1$

$\Rightarrow bdx^2 = -acy^2$ ....(5)

(1)−(3) $\Rightarrow \left(\frac{1}{a}-\frac{1}{c}\right)x^2 + \left(\frac{1}{b}-\frac{1}{d}\right)y^2 = 0$

$\Rightarrow \frac{c-a}{ac}x^2+\frac{d-b}{bd}\times\left(\frac{-bd}{ac}\right)x^2=0$ (using 5)

$\Rightarrow (c-a)-(d-b) = 0$

$\Rightarrow c - a = d - b$

$\Rightarrow c - d = a - b$

16. $\lim_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+......+\frac{1}{n}}{n^2}\right)^n$ is equal to :

(1) $\frac{1}{2}$

(2) $\frac{1}{e}$

(3) 1

(4) 0

Ans. (3)

Sol. It is 1^{∞} form

$$L = e^{\lim\limits_{n\to\infty}\left(\frac{1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}}{n}\right)}$$

$$S = 1+\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)+\left(\frac{1}{8}+........+\frac{1}{15}\right)$$

$$S < 1+\left(\frac{1}{2}+\frac{1}{2}\right)+\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right).........+\underbrace{\left(\frac{1}{2^P}+........+\frac{1}{2^P}\right)}_{2^P \text{ times}}$$

$$S < 1+1+1+1+.........+1$$

$$S < P+1$$

$$\therefore \quad L = e^{\lim\limits_{n\to\infty}\frac{(P+1)}{2^P}}$$

$$\Rightarrow L = e^{0} = 1$$

17. The total number of positive integral solutions (x, y, z) such that xyz = 24 is
(1) 36
(2) 45
(3) 24
(4) 30

Ans. (4)

Sol. x.y.z = 24
$x.y.z = 2^3.3^1$
Now using beggars method.
3 things to be distributed among 3 persons
Each may receive none, one or more
$\therefore\ ^5C_2$ ways
Similarly for '1' $\quad\therefore\ ^3C_2$ ways
Total ways = $^5C_2\ .\ ^3C_2$ = 30 ways

18. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point :
(1) (4, 5)
(2) (5, 4)
(3) (4, 4)
(4) (5, 5)

Ans. (4)

Sol. $$\frac{dy}{dx} = \frac{(x-2)^2+y+4}{(x-2)} = (x-2)+\frac{y+4}{(x-2)}$$

Let x – 2 = t $\Rightarrow$ dx = dt
and y + 4 = u $\Rightarrow$ dy = du

$\frac{dy}{dx}=\frac{du}{dt}$

$\frac{du}{dt}$ =t + $\frac{u}{t}\Rightarrow\frac{du}{dt}-\frac{u}{t}=t$

I.F = $e^{\int\frac{-1}{t}dt}=e^{-\ln t}=\frac{1}{t}$

u. $\frac{1}{t}=\int t.\frac{1}{t}dt\Rightarrow\frac{u}{t}=t+c$

$\frac{y+4}{x-2}$ = (x – 2) + c

Passing through (0, 0)

c = 0

$\Rightarrow$ (y + 4) = $(x-2)^2$

19. The value of $\int_{-1}^{1}x^2e^{[x^3]}dx$, where [t] denotes the greatest integer $\le$ t, is :

(1) $\frac{e+1}{3}$

(2) $\frac{e-1}{3e}$

(3) $\frac{e+1}{3e}$

(4) $\frac{1}{3e}$

Ans. (3)

Sol. I = $\int_{-1}^{0}x^2.e^{-1}dx+\int_{0}^{1}x^2dx$

$\therefore$ I = $\left.\frac{x^3}{3e}\right|_{-1}^{0}+\left.\frac{x^3}{3}\right|_{0}^{1}$

$\Rightarrow$ I = $\frac{1}{3e}+\frac{1}{3}$

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20. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is:

(1) $\frac{1}{8}$

(2) $\frac{1}{27}$

(3) $\frac{3}{4}$

(4) $\frac{3}{8}$

Ans. (1)

Sol. Probability of not getting intercepted = $\frac{2}{3}$

Probability of missile hitting target = $\frac{3}{4}$

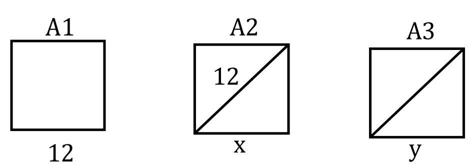
$\therefore$ Probability that all 3 hit the target = $\left(\frac{2}{3}\times\frac{3}{4}\right)^3 = \frac{1}{8}$

Section : Mathematics Section B

1. Let A_1, A_2, A_3, …… be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is ____________.

Ans. (9)

Sol.



$x = \frac{12}{\sqrt{2}}$ $\quad$ $y = \frac{12}{(\sqrt{2})^2}$

$\therefore$ Side lengths are in G.P.

$T_n = \frac{12}{(\sqrt{2})^{n-1}}$

$\therefore$ Area = $\frac{144}{2^{n-1}} < 1$ $\quad \Rightarrow 2^{n-1} > 144$

Smallest n = 9

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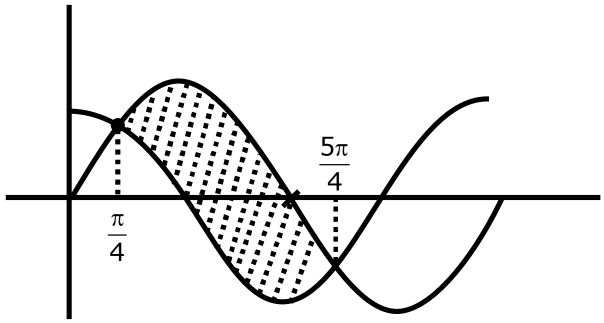
2. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to ________

Ans. (64)

Sol.



$$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\, dx = [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$$

$$= -\left[\left(\cos\frac{5\pi}{4} + \sin\frac{\pi}{4}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$$

$$= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\Rightarrow A^4 = \left(2\sqrt{2}\right)^4 = 64$$

3. The locus of the point of intersection of the lines $(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}x - y - 4(\sqrt{3})$ k = 0 is a conic, whose eccentricity is ________.

Ans. (2)

Sol. $\sqrt{3}kx + ky = 4\sqrt{3}$ .... (1)

$\sqrt{3}kx - ky = 4\sqrt{3}k^2$ ....(2)

Adding equation (1) & (2)

$2\sqrt{3}kx = 4\sqrt{3}(k^2 + 1)$

$x = 2\left(k + \frac{1}{k}\right)$ ....(3)

Substracting equation (1) & (2)

$y = 2\sqrt{3}\left(\frac{1}{k} - k\right)$ ....(4)

$\therefore \frac{x^2}{4} - \frac{y^2}{12} = 4$

$\frac{x^2}{16} - \frac{y^2}{48} = 1$ Hyperbola

$\therefore e^2 = 1 + \frac{48}{16}$

e = 2

4. If A = $\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1}$

$= \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to _______.

Ans. (13)

Sol. $A = \begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$

$\Rightarrow I + A = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$\Rightarrow I - A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ $\{\therefore |I - A| = \sec^2\theta/2\}$

$\Rightarrow (I - A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}} \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$\Rightarrow (I+A)(I-A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}} \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$= \frac{1}{\sec^2\frac{\theta}{2}} \begin{bmatrix} 1-\tan^2\frac{\theta}{2} & -2\tan\frac{\theta}{2} \\ 2\tan\frac{\theta}{2} & 1-\tan^2\frac{\theta}{2} \end{bmatrix}$

$a = \frac{1-\tan^2\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$

$$b=\frac{2\tan\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$$

$\therefore\ a^2 + b^2 = 1$

5. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x = −1 and x = 1. If $\lim\limits_{x\to 0}\frac{f(x)}{x^3}=1$, then 5.f(2) is equal to __________

Ans. (144)

Sol. $f(x) = x^6 + ax^5 + bx^4 + x^3$

$\therefore\ f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

Roots 1 & − 1

$\therefore\ 6 + 5a + 4b + 3 = 0\ \&\ -6 + 5a - 4b + 3 = 0$ solving

$a = -\frac{3}{5}$ $\qquad$ $b = -\frac{3}{2}$

$\therefore\ f(x) = x^6-\frac{3}{5}x^5-\frac{3}{2}x^4+x^3$

$\therefore\ 5.f(2) = 5\left[64-\frac{96}{5}-24+8\right]=144$

6. The number of points, at which the function $f(x) = |2x + 1| -3|x+2|+|x^2 + x-2|$, $x \in R$ is not differentiable, is _________.

Ans. (2)

Sol. $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$

$$f(x)=\begin{cases} x^2-7\ ; & x>1 \\ -x^2-2x-3\ ; & -\frac{1}{2}<x<1 \\ -x^2-6x-5\ ; & -2<x<\frac{-1}{2} \\ x^2+2x+3; & x<-2 \end{cases}$$

$$\therefore\ f'(x)=\begin{cases} 2x\ ; & x>1 \\ -2x-3\ ; & -\frac{1}{2}<x<1 \\ -2x-6; & -2<x<\frac{-1}{2} \\ 2x+2; & x<-2 \end{cases}$$

Check at 1, −2 and $\frac{-1}{2}$

Non. Differentiable at x = 1 and $\frac{-1}{2}$

7. If the system of equations

$kx + y + 2z = 1$

$3x - y - 2z = 2$

$-2x - 2y - 4z = 3$

has infinitely many solutions, then k is equal to __________.

Ans. (21)

Sol. D = 0

$$\Rightarrow \begin{vmatrix} k & 1 & 2 \\ 3 & -1 & -2 \\ -2 & -2 & -4 \end{vmatrix} = 0$$

$\Rightarrow$ k (4 – 4)– 1(–12 –4) +2(–6 –2)

$\Rightarrow$ 16 – 16 = 0

Also. $D_1 = D_2 = D_3 = 0$

$$\Rightarrow D_2 = \begin{vmatrix} k & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow$ k(–8+6)–1(–12–4)+2(9+4)=0

$\Rightarrow$ –2k + 16 + 26 = 0

$\Rightarrow$ 2k = 42

$\Rightarrow$ k = 21

8. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r}.\vec{b} = 0$, then $\vec{r}.\vec{a}$ is equal to ________

Ans. (12)

Sol. $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$

$\vec{r} \times \vec{a} - \vec{c} \times \vec{a} = 0$

$(\vec{r} - \vec{c}) \times \vec{a} = 0$

$\therefore\ \vec{r} - \vec{c} = \lambda\vec{a}$

$\vec{r} = \lambda\vec{a} + \vec{c}$

$\vec{r}.\vec{b} = \lambda\vec{a}.\vec{b} + \vec{c}.\vec{b} = 0$

$\Rightarrow \lambda$ (1–2) + 2 = 0

$\Rightarrow \lambda$ = 2

$\therefore\ \vec{r} = 2\vec{a} + \vec{c}$

$\vec{r}.\vec{a} = 2|\vec{a}|^2 + \vec{a}.\,\vec{c}$

= 2(1 + 4 + 1) + (1 – 2 + 1)

= 12

9. Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that $x + y + z > 0$ and $xyz = 2$.

If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is _______.

Ans. (7)

Sol. $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$ $\quad \therefore |A| = \left(x^3 + y^3 + z^3 - 3xyz\right)$

$A^2 = I_3$

$|A^2| = 1$

$\therefore (x^3 + y^3 + z^3 - 3xyz)^2 = 1$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 1$ only as $(x+y+z>0)$

$\Rightarrow x^3 + y^3 + z^3 = 6 + 1 = 7$

10. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4,5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5 is ________.

Ans. (32)

Sol. $\square\square\square$ divisible by $\rightarrow 3$

$12 \rightarrow 3, 4, 5 \rightarrow 3! = 6$

$15 \rightarrow 2, 3, 4 \rightarrow 3! = 6$

$24 \rightarrow 1, 3, 5 \rightarrow 3! = 6$

$42 \rightarrow 1, 2, 3 \rightarrow 3! = 6$

24

divisible by 5

$\square\square\boxed{5} = 12$

4×3

Required No. = 24 + 12 − 4 = 32

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